Further Pure 1 Past Paper Questions Pack A: Mark Scheme

Taken from MAP1, MAP2, MAP3, MAP4, MAP6

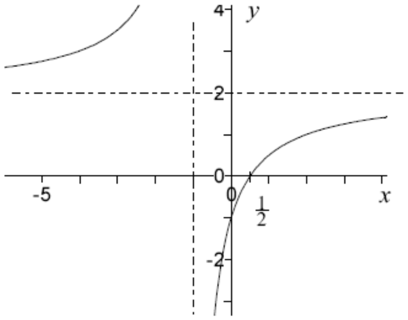
Parabolas, Ellipses and Hyperbolas

Pure 3 June 2002

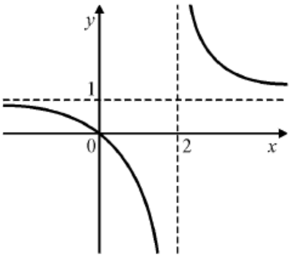
3 (a)	$x = 2 \quad y = \pm \frac{5\sqrt{5}}{3} = \pm 3.73$	M1A1	2	allow ± 3.7, or any correct numerical form

Rational Functions and Asymptotes

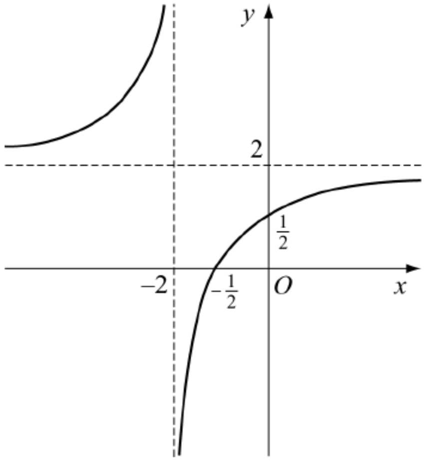
Pure 2 June 2001

Q	Solution	Marks	Total	Comments
5 (a)		B1		Asymptote at $x = -1$
		B1		Asymptote at $y = 2$
		B1		$x = \frac{1}{2}$ <u>and</u> $y = -1$
		B1	4	Generally correct: award if $y = 2$ missing but reasonable rectangular hyperbola
(b)	Solve $\frac{2x-1}{x+1} = 5$	M1A1		
	$\Rightarrow x < -2$	A1		
	and $x > -1$ from graph	B1√	4	ft on 'reasonable' graph
	Total		**8**	

Pure 2 June 2002

3		B1		Discontinuity at $x = 2$
		B1		y values $\to 1$ as $x \to \pm\infty$
		B1		Through $(0,0)$
		B1		Fully correct Condone omission of 1 and 2 on graph
	$x = 2$ <u>and</u> $y = 1$	B1	(5)	Both correct. Accept if <u>labelled</u> on the graph
	Total		**(5)**	

Pure 2 June 2003

Q	Solution	Marks	Total	Comments
7 (a)	$\begin{array}{r} 2 \\ x+2\overline{)2x+1} \end{array}$	M1		Any valid method attempted
	$\begin{array}{r} \underline{2x+4} \\ -3 \end{array}$	A1		for 2
	$\therefore \frac{2x+1}{x+2} = 2 - \frac{3}{x+2}$	A1F	3	for -3
(b)		B1 B1 B1 B1	4	One asymptote; ft $y = A$ Other asymptote Full general shape Intersections with both axes labelled (i.e. $\left[0, \frac{1}{2}\right]$ and $\left[-\frac{1}{2}, 0\right]$)

Complex Numbers / Roots of Quadratic Equations

Pure 4 June 2004

Q	Solution	Marks	Total	Comments
1(a)	$(3-\mathrm{i})^2 = 9-6\mathrm{i}+\mathrm{i}^2 = 8-6\mathrm{i}$	B1	1	
(b)(i)	$a(8-6\mathrm{i})+b(3-\mathrm{i})+10\mathrm{i}=0$	M1		Substituting 3 – i into quadratic.
	Equating R & I parts	M1A1		
	$8a+3b=0$			
	$-6a-b+10=0$			
	Attempt to solve	M1		
	$a=3, \quad b=-8$	A1A1F	6	$a=3$ is AG If $a = 3$ is assumed, allow M1A1 for b
(ii)	Sum of roots $= -\frac{b}{a}$	M1		If sum of roots is – 8 give M0
	or product $= \frac{c}{a}$			
	$\beta = -\frac{1}{3}+\mathrm{i}$	A1A1F	3	A1 for $-\frac{1}{3}$, A1 for + i
	Total		**10**	

Pure 2 June 2001

2	$\alpha+\beta=5, \quad \alpha\beta=3$ seen or $\Rightarrow$	M1		Ignore sign on sum
	New sum and product:			Alternatives:
	$\alpha+\beta+2 \qquad (\alpha+1)(\beta+1)$	M1		1. $x \mapsto x-1$ M1
	$=7 \qquad\qquad =9$	A1√		sub M1A1 result A1
	leading to $x^2-7x+9=0$	A1√	4	2. Finding roots M1A1 sub new roots M1 CAO A1
	Total		**4**	

Pure 2 June 2003

2(a)	$\alpha\beta = 2$	B1	1	$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ if seen anywhere
(b)(i)	$\alpha+\beta=-p$	B1	1	
(ii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1		correct use of $(\alpha+\beta)^2-2\alpha\beta$
	$=p^2-4$	A1F	2	ft from their $(\alpha+\beta)$and $\alpha\beta$
(c)	$p^2-4=5 \Rightarrow p=\pm 3$	A1F	1	No ft from $\alpha^2+\beta^2=(\alpha+\beta)^2$
	Total		**5**	

Pure 2 Jan 2004

Q	Solution	Marks	Total	Comments
1 (a)(i)	$\alpha\beta = \frac{1}{2}$	B1		
(ii)	$\alpha + \beta = 3$	B1	2	
(b)(i)	$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = 2$	B1√	1	
(ii)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = 6$	M1A1√	2	
(c)	$x^2 - (sum)x + (product) = 0$ $x^2 - 6x + 2 = 0$	M1 A1√	2	Replace x by $\frac{1}{x}$ $2\left(\frac{1}{x}\right)^2 - 6\left(\frac{1}{x}\right) + 1 = 0$ $\frac{2}{x^2} - \frac{6}{x} + 1 = 0$ × by x^2 to give $x^2 - 6x + 2 = 0$
	Total		**7**	

Numerical Methods

Pure 1 June 2001

3	a	Reasonable sketch of cos	B1		OE sketches
		One pt of int ⇒ one root	E1	2	AG
	b	Use of tan = sin/cos	M1		
		$f(\alpha) = 0$	A1	2	or $f(x) = 0$; convincingly shown (AG)
	c	$f(0.8) \approx -0.22036 \approx -0.220$	B1		AG: more DP shown or f(0.9) correct
		$f(0.9) \approx 0.14905 \approx 0.149$	B1	2	Allow AWRT 0.149
	d	Complete linear interpolation	M1		using neg and pos values from (c)
		$\alpha \approx 0.86$	A1	2	Allow AWRT 0.86
				8	

Pure 1 Jan 2002

5 (a)(i)	$f(1) \approx -0.443, f(1.2) \approx 0.172$	B1		numerical values needed, to at least 1DP
	Change of sign, hence root between	E1	2	sign change OE must be mentioned
(ii)	$f(1.1) \approx -0.235, f(1.15) \approx -0.0655$	M1		both attempted, not necessarily accurately
	Root between 1.15 and 1.2	A1	2	answer must be an interval, not a single value

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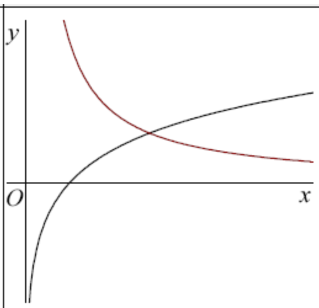
Q	Solution	Marks	Total	Comments
1 (a)	Calculation of $f(1.2)$ and $f(1.3)$	M1		where $f(x) = x^4 - (5 - 2x)$; OE
	$f(1.2) \approx -0.53,\ f(1.3) \approx 0.46$	A1		OE; accept 1 DP
	Clear justification of result	E1	3	AG: must mention sign change OE
(b)	$f(1.25) \approx -0.06$	B1		OE; accept –0.1
	Root nearer to 1.3	B1F	2	ft wrong value
	Total		**5**	

Pure 1 Jan 2003

2 (a)	$x^3 = x + 1 \Rightarrow x^3 - x - 1 = 0$	B1	1	Convincingly shown (AG)
(b)(i)	$f(1.2) = -0.472,\ f(1.4) = 0.344$	B1B1		OE; Numerical values needed
	Sign change implies root between	E1	3	Sign change OE must be mentioned
(ii)	Attempt at $f(1.3)$ $(= -0.103)$	M1		
	Root between 1.3 and 1.4	A1		PI
	$f(1.35) = 0.110375$, so root between 1.3 and 1.35	M1	3	Allow good attempt leading to values differing by 0.05
(iii)	$\alpha \approx 1.3$	A1	1	
	Total		**8**	

Pure 2 June 2001

7 (a)	 Graph $\ln x$	B1		
	Graph $\frac{3}{x}$	B1	2	
(b)(i)	$f(3)>0$ $f(2)<0$ $\Rightarrow$ root in $2<x<3$	M1A1	2	
(ii)	$f'(x)=\frac{1}{x}+\frac{3}{x^2}$	B1		
	Use of Newton-Raphson formula	M1A1√		
	$x_1 = 2.82$	A1	4	AWRT (3 s.f) is OK
	Total		**8**	

Pure 2 June 2003

Q	Solution	Marks	Total	Comments
4 (a)	Let $f(x)=2\cos x-\frac{1}{x}$			
	$f(0.6)=-0.016$	M1		use of calculator in radian mode
	$f(0.7)=0.101$	A1		
	Change of sign indicates a root of $f(x)=0$ between 0.6 and 0.7	E1	3	
(b)	$f'(x)=-2\sin x+\frac{1}{x^2}$	M1		attempt at differentiation
		A1		CAO
	$x_2=0.6-\frac{f(0.6)}{f'(0.6)}$	M1		Use of Newton-Raphson
	$=0.6-\frac{-0.0160}{1.6485}$	m1		$f(0.6)$ correct **and** their $f'(0.6)$ attempted
	≈ 0.610	A1F	5	ft on their $f'(x)$
	Total		**8**	

Pure 2 Jan 2004

Q	Solution	Marks	Total	Comments
6 (a)	$f(1)=0.341$ $f(2)=-0.091$	M1		
	Change of sign $\Rightarrow$ $\therefore$ root in the interval $1\le x\le 2$	A1	2	
(b)(i)	$f'(x)=\cos x-\frac{1}{2}$	B1	1	
(ii)	$x_{n+1}=x_n-\frac{f(x)}{f'(x_n)}=x_n-\frac{\sin x_n-\frac{1}{2}x_n}{\cos x_n-\frac{1}{2}}$	M1		N-R formula used
	$x_0=2 \quad \therefore \quad x_1=2-\frac{\sin 2-1}{\cos 2-\frac{1}{2}}$	m1		Radians used in correct formula
	$x_1=1.901\approx 1.9$	A1	3	AG

Pure 3 June 2001

5	x	y	step x	$\frac{dy}{dx}$	step y			
	0	3	0.5	3	1.5	M1A1		M1 use $\partial y=\frac{dy}{dx}\partial x$
	0.5	4.5	0.5	2.958	1.479	M1A1		accept $y=1.48$
	1.0	5.979						
		5.98				A1	5	CAO
					Total		**5**	

Pure 3 Jan 2002

3 (a)	x	y	$\frac{dy}{dx}$	dx	dy			
	-2	1	-0.5	0.5	-0.25	M1A1		
	-1.5	0.75	-0.333	0.5	-0.167	M1		
	-1	0.583						
	0.58					A1	4	
(b)	Reduce the step size					B1	1	CAO
					Total		**5**	

Pure 3 Jan 2003

Q	Solution					Marks	Total	Comments
4	$\frac{dy}{dx}=\sqrt{x^2-5}$							Clarification of marks:
	x	y	$\frac{dy}{dx}$	dx	dy	M1		M1 calculate $\frac{dy}{dx}$; use result $\times 0.5=dy$
	3	1	2	0.5	1	A1		A1 $dy=1$
	3.5	2	2.69	0.5	1.346	M1		M1 $y\to y+dy;\ x\to x+dx;$ calculate $\frac{dy}{dx}$; use result $\times 0.5=dy$
	4	3.346				A1		
	3.35					A1	5	A1 $y=2$ $dy=1.346$ (allow 1.35) A1 $y=3.35$ CAO
					Total		**5**	

Pure 3 June 2003

Q	Solution					Marks	Total	Comments
6(a)	t	x	$\frac{dx}{dt}$	dt	dx			
	0	1	1.8	0.3	0.54	M1		
						A1		Allow M1A1 with $dx = 0.3\begin{cases} dt = 0.54 \\ \frac{dx}{dt} = 1.8 \end{cases}$
	0.3	1.54	1.692	0.3	0.5076	M1		(but 2 / 4 max)
	0.6	2.0476				A1	4	AWRT 2.05

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2	x	y	step x	$\frac{dy}{dx}$	step y			$\frac{dy}{dx} = 0.5$ M1
	1	0.5	0.25	0.5	0.125	M1A1		Step dy = 0.125 A1
	1.25	0.625	0.25	0.3386	0.0846	M1A1		1.25; step y + 0.5; step $y = 0.25\frac{dy}{dx}$ M1
	1.5	0.7096						
			$x = 0.71$			A1	5	0.08 (46) AWRT A1
	Total						**5**	

Matrix Transformations

Pure 6 Jan 2002

Q	Solution	Marks	Total	Comments
4 (a)	Rotation, $\frac{\pi}{6}$, anticlockwise	B1B1B1	3	
(b)	$\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	B3	3	B2 if 2 correct
(c)(i)	$\mathbf{M}_1\mathbf{M}_2$ considered	M1		
	$\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$	A1	2	
(ii)	Reflection	B1		
	Line at 75° to x-axis	B2	3	
	Total		**11**	

Pure 6 Jan 2003

Q	Solution	Marks	Total	Comments
1 (a)	$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$	M1A1	2	
(b)	$\begin{bmatrix} 1 & * \\ 2 & * \end{bmatrix}$	B1		
	$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$	M1A1	3	
	Total		**5**	

Pure 6 June 2003

2 (a)	$\mathbf{M}$ is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$			
	or	B1		Explain and justify $\frac{\pi}{3}$
	where $\sin\theta = \frac{\sqrt{3}}{2}$, $\cos\theta = \frac{1}{2}$			
	$\therefore$ **M** represents			
	a rotation anticlockwise about O	B1		
	of $\frac{1}{3}\pi$	B1	3	condone 60° (if stated about the x-axis B0)
(b)	$6 \times \frac{\pi}{3} = 2\pi \qquad \therefore \mathbf{M}^6 = \mathbf{I}$	M1A1	2	
	Total		**5**	